Predictive Functional Controller (PFC) with Novel Observer Method for Pneumatic Positioning System

A.R Azira, Khairuddin Osman, S. I. Samsudin, Siti Fatimah Sulaiman
Faculty of Electronic and Computer Engineering,
Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka
khairuddin.osman@utem.edu.my

Abstract—Nowadays, the pneumatic system is more complex which leads to the development of an intelligent pneumatic system. Due to the difficulties in controlling the position and force of pneumatic actuators nonlinearities existed. This paper proposes a design of Predictive Functional Control (PFC) using two different types of observers such as full-order and reduced order as a novel method to come out with these issues. The mathematical model of the pneumatic system come from System Identification (SI) method and third order Auto-Regressive with Exogenous Input (ARX) has been chosen as a model structure. Matlab/Simulink has been utilized as the platform and the performance of the controller using both observers have been validated in simulation and real-time experiment. The comparison has been made to identify which observers are more efficient by taking into account the value of Steady State Error ($Sse$), Percentage of Overshoot ($%O_s$), Settling Time ($Ts$) and Rise Time ($Tr$). Real-time experiment results show that the strategy using reduced-order observer is more efficient because this strategy can reduce more $Sse$.

Index Terms—Predictive Functional Controller (PFC); Auto-Regressive with Exogenous Input (ARX); Full-Order Observer; Reduced-Order Observer; Intelligent Pneumatic Actuator (IPA).

I. INTRODUCTION

Many applications in Mechatronics, actuators that can process information from input given and control the output independently are highly in demand [1]. The pneumatic actuating system gives more advantages of high power-to-weight ratio, lightweight, comparatively low cost, easier maintenance, and having simpler structure compared to other actuators [2]. Pneumatic systems also used to overcome their nonlinearities which are high friction forces, dead band and dead time due to the compressibility air [3]. However, it is very difficult to control the nonlinear characteristics, positions, force and pressure [4]. Many developments have been tested to the pneumatic actuators to analyze the different automation and industrial purposes depend on desired accuracy and performance and the amount of force that suitable for each particular application [5]. Many strategies have been used before an where this researcher was designed comparison with PI, PFC and PFC-O [4] and other researcher also designed PID [6]. However, PFC and PFC-O PFC and PFC-O give better results than PI. This is because a PFC gives a faster response with 0% overshoot.

There have many research has been combined PFC with Observer. In this study, the use of observer or state estimator is very essential especially to validate the strategy in a real-time environment. The observer is used to estimating the internal states of the pneumatic system for the purpose of PFC. Full order and reduced order are the common type of observer used. The observer used to calculate states are not measurable by using the values of the current output of the plant $y(ki)$ and the current value of the control signal $u(ki)$. Full-order observer estimates all state variable where reduce-order observer estimates only unmeasured state variable. In this study was used in previous research where the researcher used PFC with the full-order observer (PFC-O) to control pneumatic system[4]. The researcher has compared the result in simulation and real time. The results showed that PFC-O controller gives better control performance compared to the controller without an observer. PFC-O was designed to have the ability to estimate the states in real time experiment. This research takes the initiative from this research to compare PFC with two type observers which are a full-order observer and reduced-order observer.

This paper is organized as follows, the methodology will be explained in section II, controller design in part III, results and discussion in part IV, end with conclusion and references.

II. METHODOLOGY

A. System Modelling

The pneumatic actuator system used in this study is a linear double-acting type and new Intelligent Pneumatic Actuator (IPA) was developed by Ahmad ’Athif Mohd Faudzi [7] to overcome the limitations of the actuator. Figure 1 shows five main components of IPA system which are: 1) Laser stripe code with position accuracy of 0.169 mm and position accuracy of 0.01 mm and the actuator is 200 mm stroke and also can deliver maximum force up to 120 N, 2) KOGANEI-ZMAIR optical sensor was used capable of detecting smaller pitch of 0.01 mm, 3) Pressure Sensor was used to check the pressure in chamber during performing the control action of cylinder, 4) Valve represent as Pulse-Width Modular (PWM) signal will control the inlet and outlet air of the cylinder and 5) Programmable System on Chip (PSoC) microcontroller act as brain to control system and to performs local control to suit the requirements or any related applications [8].

Figure 2 shows the schematic diagram of IPA systems inlet while valve 2 will control the air outlet (exhaust) [4]. The Linear actuator will control by two air inlets with air pressure 0.6MPa and one exhausted outlet. By supplying constant air pressure to chamber 1, air will regulate in chamber 2. The movement of the actuator to let and right can be controlled by...
manipulating pressure in chamber 2 only. The pressure sensor was connected to PSOC to take the value of pressure data.

Figure 1: IPA Systems

The uniqueness of this system compare to other pneumatic system available in the market is, the movement of the stroke is only control using one chamber by controlling air inlet in chamber 2 only rather than controlling both chambers mechanism. The mathematical model of the system is obtained using System Identification (SI) method. 2000 measurements consist of input and output data with 0.01s sampling time was used for this purpose. Third order linear Auto-Regressive with Exogenous Input (ARX) was used to represent the real system in this study. The discrete transfer function of the linear third order ARX can be represented as equation (1) and the discrete state space as equation (2).

\[
B_{\text{Position}}(Z^{-1}) = \frac{1}{{1 - 1.932Z^{-1} + 0.1577Z^{-2}}} \\
A_{\text{Position}}(Z^{-1}) = \frac{1}{{1 - 1.932Z^{-1} + 0.109Z^{-2} - 0.0003498Z^{-3}}}
\]

(1)

where A,B,C,D matrix

\[
A = \begin{bmatrix} 1.9320 & 1 & 0 \\ -1.09 & 0 & 1 \\ -0.1577 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0013 \\ 0.0005 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D = 0
\]

(2)

B. Predictive Functional Control

In this research PFC was approached as pneumatic controller strategy. PFC was designed based on the state-space form of the plant because of easy generalization to multivariable systems, easy analysis of the closed-loop properties and allowance for online computation [9]. In this section, the pneumatic model as in Equation (1) was converted into the state-space form. Equation (3) is a general discrete state-space model.

\[
x_{k+1} = Ax_k + Bu_k
\]

(3)

\[
y_k = Cx_k + Du_k
\]

(4)

For prediction with a strictly proper system, \( D = 0 \)

\[
x_{k+1} = Ax_k + Bu_k
\]

(5)

\[
y_k = Cx_k
\]

(6)

Convert the state-space model into state prediction equation

\[
\begin{bmatrix} x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ \vdots \end{bmatrix} = \begin{bmatrix} A & A^2 & A^3 & \cdots \\ B & AB & A^2B & \cdots \\ A^2B & AB & A^2B & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_k \\ u_k \\ u_{k+1} \\ u_{k+2} \\ \vdots \end{bmatrix}
\]

(7)

and output prediction equation:

\[
\begin{bmatrix} y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ \vdots \end{bmatrix} = \begin{bmatrix} CA & CA^2 & CA^3 & \cdots \\ CAB & CB & \cdots \\ CA^2B & CB & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_k \\ u_k \\ u_{k+1} \\ u_{k+2} \\ \vdots \end{bmatrix}
\]

(8)

This arrangement can be achieved by introducing the prediction matrices, \( P \) and \( H \). Therefore, the model used is a linear as shown in Equation (9) and Equation (10)

\[
x_k = P_{xx}x_k + H_{xx}u_{k-1}
\]

(9)

\[
y_k = Px_k + Hu_{k-1}
\]

(10)

\( x_k \) is the state model where \( u_k \) is the input model, \( y_k \) is the measured output model. \( P_{xx}, H_{xx}, P \) and \( H \) are matrices and vectors of the right dimension respectively. PFC starting point of formulating control law is developing the reference trajectory equation that can be done by placing the desired closed-loop dynamic into the reference trajectory. Given if the actual set point is \( r \), and the loop set point, \( w \) is a first order lag, where \( w \) calculated by the following equation

\[
w_{k+i/k} = r_k - (r_k - y_k)w^i
\]

(11)

where \( i \) is the value of \( n \), \( y_k \) is the most recent measured output and \( \Psi (0 < \Psi < 1) \) is scalar the time constant and a tuning parameter setting the desired closed-loop poles. Equation (11) is the predictive essence of control the strategy. This is to have the set point trajectory closely following the reference desired closed-loop behavior. In addition, it must also deal with the set of coincidence points. This can be achieved by using the Degree of Freedom (DOF) to force the equality of the prediction and the reference trajectory at a number of points. Therefore, solving the control moves is:
\( y_{k+n} = w_{k+n} \)  

where \( n = n1, n2 \ldots \). These equalities are called coincidence points. In normal cases, there are no more than two coincidence points. In this research, focus only on one coincidence point, \( n1 \). Thus, at a single coincidence point and using Equation (11) and (12), the control law can be determined by:

\[
y_{k+n} = w_{k+n} = r_k - (r_k - y_k)\psi^i
\]

Hence, substituting Equation (9) and (10) into (12);

\[
y_{k+n} = P x_k + H u_{k-1} = r_k - (r_k - y_k)\psi^i
\]

Assuming \( u_{k+i} = u_k \), thus the control law can be formulated by rewriting Equation (13) and obtain;

\[
u_k = -H^{-1}[P x_k + (r_k - y_k)\psi^i] \quad \text{(14)}
\]

\[
u_k = -K_c x_k + P_c r_k \quad \text{(15)}
\]

where \( K_c = -H^{-1}(P - \psi^i y_k) \) and \( P_c = -H^{-1}(1 - \psi^i) \). Now, the prediction algorithm can easily be recognized from the fixed linear feedback law. Thus, the typical posterior stability and sensitivity analysis can be easily achieved in a straightforward manner.

As stated earlier, According to Rossiter [9], there is only one coincidence point. The typical procedure with one coincidence point would be as follows:

i. Choose the desired time constant, \( \psi \).
ii. Do search for coincidence horizon, \( n1 = l, 2 \ldots \) large and find the associated control law for each \( n1 \).
iii. Select the \( n1 \), which gives closed-loop dynamics closest to the chosen \( \psi \).
iv. Simulate the proposed law. Otherwise, reselect \( \psi \) and go to step 2.

The optimal parameter tuning is an optimization problem, which requires implementation of global optimization strategy such as Particle Swarm Optimization (PSO). However, it is still possible to find an optimized parameter by selecting the parameter value with an increment of 0.05.

C. Observer

A design of observer is essential in order to estimate the state of the pneumatic system model. A state observer will provide an estimation of the internal state of a given system, from measurements of the input and output of the system. Assuming that discrete-time state-space model system is:

\[
\dot{x}(k+1) = A \dot{x}(k) + Bu(k) \quad \text{(16)}
\]

\[
y = Cx \quad \text{(17)}
\]

By using feedback estimation error term \( y - C\dot{x}(k) \) as a correction term. By substitute this in equation (16) this equation will represent full-order observer.

\[
\dot{x}(k+1) = A \dot{x}(k+1) + Bu(k) + K_{ob}(y - C\dot{x}) \quad \text{(18)}
\]

In this research PFC will implement with full order and Reduced-order observer. An observer must be designed as the state variable \( x(k) \) at time \( k \) is not measurable [4]. The function of the observer is to calculate the future state by using the values of the current output of the plant \( y(k) \) and the current value of the control signal \( u(k) \). Reduced-order observer is a system that estimates the components of the state that cannot be directly reconstructed from the output. PFC with reduced order observer is established to reduce steady state error (Sse), based on estimate value of alpha and system output. To develop the deference equation for the reduced order observer, let us first divide the state vector to \( x_a(k) \), which is the known or the measured state, and \( x_b(k) \), which is the unknown state or the state to be estimated.

\[
x(k) = \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix}
\]

With that, equation (16) and (17) can be written as,

\[
\begin{bmatrix} x_a(k+1) \\ x_b(k+1) \end{bmatrix} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} u(k) \quad \text{(20)}
\]

\[
y(k) = [1 \ 0 \ldots 0] \begin{bmatrix} x_a(k) \\ x_b(k) \end{bmatrix} \quad \text{(21)}
\]

Equation (20) can be divided into the known equation,

\[
x_a(k+1) = A_{aa}x_a(k) + A_{ab}x_b(k) + B_a u(k) \quad \text{(22)}
\]

\[
x_a(k+1) = A_{aa}x_a(k) + A_{ab}x_b(k) + B_a u(k) \quad \text{(23)}
\]

and to the estimated states equation,

\[
x_b(k+1) = A_{ba}x_a(k) + A_{bb}x_b(k) + B_b u(k) \quad \text{(24)}
\]

By comparing the states equation (16) with the estimate state equation (24)

\[
x(k+1) = Ax(k) + Bu(k) \quad \text{(25)}
\]

And also by comparing known states equation (17) with the output equation (23)

\[
y(k) = Cx(k) \quad \text{(26)}
\]

Then the reduced order observer is then developed by substituting the following

\[
x_b(k) \rightarrow x(k) \quad A_{bb} \rightarrow A
\]

\[
[Abax_a(k) - B_b u(k)] \rightarrow Bu(k)
\]

\[
[x_a(k+1) - A_{aa}x_a(k) - B_a u(k)] \rightarrow y(k) \quad A_{ab} \rightarrow C
\]
By making the previous substitutions into the full-order observer equation (18)
\[
\dot{x}(k + 1) = A\hat{x}(k) + Bu(k) + K_0b[y(k) - C\hat{x}(k)]
\]
\[
\dot{x}_b(k + 1) = (A_{bb} - K_0bA_{ab})\hat{x}_b(k) + K_0b[x_a(k) - A_{a}u(k)] + A_{ba}x_a(k) + B_{b}u(k)
\]  
(27)

From equation (21), we have,
\[
y(k) = x_a(k)
\]

By substituting into equation (18), yields the reduced order observer difference equation
\[
\dot{x}_b(k + 1) = (A_{bb} - K_0bA_{ab})\hat{x}_b(k) + K_0b[y(k) + (A_{pa} - K_0bA_{a})y(k)] + (B_{b} - K_0bB_{a})u(k)
\]  
(28)

III. CONTROLLER DESIGN

Two types of the controller have been designed in this research which is PFC with the full-order observer and PFC with the reduced-order observer. Both designed are expected can be able to control the position of cylinder actuator. The comparison has been made to observe which controller can reduce steady stable error in the pneumatic system.

A. PFC with Full-order Observer

The observer has been designed based on the discrete state space matrices A, B and C. In this research, the inputs of the observers are the output of the plant. Where the output of observer are the estimated states and the estimated output, calculated from the estimated states multiplied with matrix C. Figure 3 shows how PFC connected into a full-order observer.

B. PFC with Reduced-order Observer

When one or more of the states can be measured, only the unknown states will be estimated. Figure 4 is a block diagram of how the PFC connected into a reduced-order observer.

IV. RESULT AND DISCUSSION

This section was discussed and observed the performance and value of SSE of two strategies which are PFC with the full-order observer and PFC with reduced-order observers. The result has been comparing between both strategies in simulation and real time experiment. Two types of control signals have been applied which are step signal to test the performances controller and multi-step signal in order to validate the performances.

A. Simulation Result

The performance of propose method has been tested via simulation before being realized in real time experiment. The simulation was carried out in MATLAB/Simulink and results are shown in Table 1. Table 1 shows the parameter value has been compared between both strategies where Figure 5 until Figure 8 shows performance response to the system. The value of alpha was tune manually from 0.90 to 1.00. The result shows that PFC with both observers has approximately 0 value of %OS and SSE. The value of TS is same for both strategy when alpha equal to 0.90 which is 0.5665. The value of TS when alpha equal to 0.90 is 0.8163 sec shows that PFC with full-order observer give 0.0003s faster respond compared to PFC with reduced-order observer where the value of TS is 0.8166. The result shows that when alpha increases the value of TS and TS also increase for both strategies. It has been taking more than 2 seconds for TS and more than 3 seconds for Tr when alpha equal to 0.99. Meanwhile the value of %OS and SSE maintain approximately 0 for both strategies. PFC with full-order observer has not much improvement or efficiency because the value is quite similar compared to PFC with the reduced-order observer.

This is because of simulation is linear which not contain the nonlinearities compared to actual systems. In term of TS value when alpha is 0.90, PFC with full-order observer better than PFC with the reduced-order observer.

<table>
<thead>
<tr>
<th>Alpha (α)</th>
<th>Reduced-order</th>
<th>Full-order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSE (mm)</td>
<td>%OS (sec)</td>
</tr>
<tr>
<td>0.90</td>
<td>0.5656</td>
<td>0.8166</td>
</tr>
<tr>
<td>0.95</td>
<td>0.6379</td>
<td>1.0493</td>
</tr>
<tr>
<td>0.96</td>
<td>0.6953</td>
<td>1.1874</td>
</tr>
<tr>
<td>0.97</td>
<td>0.8130</td>
<td>1.4394</td>
</tr>
<tr>
<td>0.98</td>
<td>1.1080</td>
<td>2.0030</td>
</tr>
<tr>
<td>0.99</td>
<td>2.1909</td>
<td>3.9114</td>
</tr>
</tbody>
</table>

Figure 5 and Figure 6 shows simulation response of PFC with the reduced-order observer and PFC with full-order observer by using step signal. The figure shows how both
strategies react to the system. The graph has been plotted time in seconds versus position (mm). Both figures clearly can be seen the amplitude of respond is 100mm reach the peak point that has been set. Peak point reaches 100% that confirm the value of Sse is 0. Figure 7 and Figure 8 using the Multi-step signal in order to validate the performance of PFC with both observers. Obviously both figures show percentage overshoot is 0% and Sse approximately to 0 is guaranteed using both strategies.

Table 2 shows that both strategies have Sse. When alpha equal to 0.90 the value of Sse is 0.45mm for PFC with reduced-order observer where Sse for PFC with full-order observer is 0.56mm which is lower 0.11mm compared to strategies using reduced-order. The rising time, Tr for reduced-order is faster than full-order where the different is 0.008 seconds. The value of percentage overshoot is approximate 0%. The value of Sse increase when alpha is an increase. When the alpha equal to 0.99 the value of Sse more than 6 mm for both strategies. The result also showed where rise time and settling time increased as alpha increased. In term of comparing the Sse and Tr PFC with reduced-order observer gives more improvement and efficiently compare to PFC with the full-order observer.

**Table 2**

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Reduced-order</th>
<th>Full-order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sse (mm)</td>
<td>%Os</td>
</tr>
<tr>
<td>0.90</td>
<td>0.45</td>
<td>0</td>
</tr>
<tr>
<td>0.95</td>
<td>1.23</td>
<td>0</td>
</tr>
<tr>
<td>0.97</td>
<td>1.87</td>
<td>0</td>
</tr>
<tr>
<td>0.98</td>
<td>3.30</td>
<td>0</td>
</tr>
<tr>
<td>0.99</td>
<td>6.48</td>
<td>0</td>
</tr>
<tr>
<td>1.00</td>
<td>100</td>
<td>∞</td>
</tr>
</tbody>
</table>

Figure 9 and Figure 10 shows real-time experiment response of reduced-order and full-order observer to PFC by using step signal. The amplitude has been set 100mm for both strategies. Figure 9 shows that the amplitude of PFC with reduced-order when alpha equal to 0.90 is 99.55mm which is closer to reach the peak point compared to amplitude PFC with full-order observer when alpha equal to 0.90 is 99.44mm. Both figures clearly showed that when alpha increases the amplitude is decrease. This is because Sse will increase when alpha in the increase. The strategies using full-order observer have more Sse compared to strategies using reduced-order observer. Figure 11 and Figure 12 using the Multi-step signal in order to validate the performance of PFC with both observers. There is no overshoot produced using the proposed strategy and the steady-state error approximately to 0 is guaranteed using both strategies.

**B. Real-time Experiment**

Table 2 shows real time experiment performance respond for PFC with the reduced-order observer and PFC with full-order observer into the systems. In the real time experiment
V. CONCLUSION

This paper proposes a design of Predictive Functional Control (PFC) using two different types of observers in order to come out with these issues. Both observers have been designed with PFC. The performance of both control strategies has been tested in simulation and real-time experiment. Both strategies are capable of controlling the system well. However PFC with reduced-order observer was produced the best value $Sse$ which is 0.45 in real time experiment. By taking result were conclude that strategy using reduced-order give the best performance in terms of $Sse$, rising time and settling time. Future work will study Particle Swarm Optimization algorithm technique to get the best value of alpha in PFC. The best value of alpha will implement into the system.

ACKNOWLEDGMENT

The authors would like to thanks Universiti Teknikal Malaysia Melaka (UTeM), Faculty of Electronic and Computer Engineering (FKEKK) and Centre for Telecommunication Research and Innovation (CeTRI) for their support.

REFERENCES