Fuzzy Analytic Hierarchy Process (FAHP) Integration for Decision Making Purposes: A Review

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ABSTRACT: Fuzzy Analytic Hierarchy Process (FAHP) is generally used as decision making purpose. However, AHP provides a single numerical number. As the method can only yield binary decisions of either “yes” or “no”, the method is not compatible when the decision has uncertainties. Consequently, previous researchers have fused AHP with fuzzy to process alternatives with uncertainty judgment. In this paper, fundamental knowledge of separate tool was reviewed. Fundamental of both fuzzy and AHP were also summarized in this article. The next phase of this paper was to split the types of FAHP namely to determine the relative weight and the rank or score. Two methods were utilized under FAHP to determine relative weight; triangular and trapezoidal FAHP. Next type of FAHP was to determine the score of each alternative often carried out after triangular or trapezoidal FAHP. The second types of FAHP were Fuzzy TOPSIS, and VIKOR. These FAHPs were explained and steps to carry out were presented in this paper. In conclusion, all types of FAHP are compared in terms of computational time, number of steps and level of difficulty.

KEYWORDS: FAHP; Trapezoidal FAHP; Fuzzy TOPSIS; Fuzzy VIKOR

1.0 INTRODUCTION

This article analyzes the concepts of Analytic Hierarchy Process (AHP) and Fuzzy Logic to solve different types of problems in industrial and manufacturing system [1]. The AHP hierarchy model enables decision makers to break a master problem into smaller sub problems to be solved categorically as proven by many studies. Previous researchers have carried out experiment using hybrid fuzzy AHP in manufacturing industries [3], electrical field, medical, economic survey
and other relevant fields. The aim of this article was to review variation of Fuzzy AHP methods. Besides, the advantage of AHP is that when a change is made in the upper level. The steps are too complex and tedious, making the user take longer time to finish up AHP [5]. It is proven to be inconvenient when the scaling needs to be changed to suit certain conditions resulting from computation of consistency ratio and sensitivity analysis [5-6]. Furthermore, a recent study conducted [7] also proves that the method does not consider either the uncertainties or risks related to the performance of the vendor. In this article, a discussion will be made on literature concerning the usage of Fuzzy AHP in several fields. The fields that never implement Fuzzy AHP method will be ascertained and the reasons for not introducing Fuzzy AHP as problem solver will be determined.

2.0  TYPES OF FUSION BETWEEN AHP AND FUZZY

There are several methods to combine AHP with Fuzzy Logic method. Some of the fusions are Triangular AHP, Trapezoidal AHP, both Fuzzy TOPSIS and VIKOR. The use of fuzzy is to decide for linguistic judgment criteria where uncertainty is present in a problem. Fuzzy number is a subset of single real number that represents human’s judgement to present certain criteria according to class interval during their judgments [8-9]. However, this paper will not include discussion on Fuzzy MOORA, PROMETHEE, ELECTRE, DEMATEL and other techniques due to their lengthy steps and limited applications to solve industrial applications.

2.1  Fusion of Fuzzy AHP to Determine Relative Weight

There are several membership functions to obtain the weightage of alternatives. Some widely – used functions are Triangular AHP and Trapezoidal AHP.

2.1.1 Triangular Fuzzy AHP

There are six steps to perform Triangular AHP. The initial step is performing weight scale using pairwise comparison method. Whole number is used to represent superior criterion whereby reciprocal judgment is used for least important criterion [10]. The second step is to implement fuzzy analytic hierarchy process (FAHP). Detailed study by Chang [11] depicted that the basic triangular concept where the weightage of criterion is represented by using three values represented by Equations (1) and (2). The theory is consistent with the one presented in [12].
\[ a_{ij} = (l_{ij}, m_{ij}, u_{ij}) \]  

\[ a_{ij}^{-1} = \left( \frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}} \right) \]  

\[ M_{gi}^1, M_{gi}^2, ..., M_{gi}^m, i = 1, 2, ..., n \]  

Each alternative was determined using triangular fuzzy number for obtaining the goal. Hence, \( m \) is the extent analysis of values for all objects that are obtained using the following signs:

Whereby combination of all subjects from Equation (3) can be combined to be as \( M_{gi}^j (j = 1, 2, ..., m) \) indicated as triangular AHP fuzzy numbers [13]. From this form, it can be divided into 4 other steps. The third step functions as to determine the extent that fuzzy synthetic value with respect to \( i \)-th object [14]. The extent fuzzy value is presented in Equation (6). In order to carry out fuzzy summation operation, \( m \) value of extent analysis is performed by Equation (5) while the inverse form is shown in Equation (4).

\[ S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} \]  

\[ \sum_{j=1}^{m} M_{gi}^j = \left( \sum_{j=1}^{m} l_{ij}, \sum_{j=1}^{m} m_{ij}, \sum_{j=1}^{m} u_{ij} \right) \]  

\[ \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} = \left( \frac{1}{\sum_{i=1}^{n} u_{ij}}, \frac{1}{\sum_{i=1}^{n} m_{ij}}, \frac{1}{\sum_{i=1}^{n} l_{ij}} \right) \]  

The next part is identifying degree of possibilities by comparing \( M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1) \). It also can be presented in Equations (7), (8) and (9) as referred to in [15]:

\[ V(M_2 \geq M_1) = \sup \{ \min(\mu_{M_1}(x), \mu_{M_2}(y)) \}_{y \geq x} \]  

\[ V(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu_{M_1}(d) \]
In order to perform comparison between both $M_1$ and $M_2$, $V(M \geq M_1)$ with $(M \geq M_2)$ must be identified. The fifth step is to make sure that degree possibilities of fuzzy convex should be bigger than $k$ fuzzy convex [16]. Convex fuzzy is supposed to be greater in value compared to $k$ convex fuzzy. $M_i(i=1,2,3,...,k)$ is defined as:

$$V(M \geq M_1, M_2, ..., M_k) = V(M \geq M_1) \text{and}(M \geq M_2)$$

and $(M \geq M_i), i = 1, 2, 3, ..., k$ (10)

Assume $d'(A_s) = \min V(S_i \geq S_k)$ for $k=(1, 2, 3, ..., n); k \neq i$. Lastly, weight is yielded as

$$W = (d(A_1), d(A_2), ..., d(A_n))^T$$ (11)

where $A_i(i=1, 2, 3, ..., n)$ is the element presents after the computation. Final step for Triangular Fuzzy AHP is the normalization step. Equation (11) displays the sum of all elements which are divided into each object. This $W$ will be a real number which defines weight of alternatives or criterion. To conclude, this step uses three fuzzy values for comparison and is proven to be more accurate because the point considered is three rather than one point in AHP.

### 2.1.2 Trapezoidal AHP

There are four steps in performing Trapezoidal AHP. The initial step of Trapezoidal FAHP is similar to Triangular FAHP [17]. The second step is to use FAHP which consists of 4 values. The weightage criteria are represented by $x_{ij}$ as shown in Equations (12) and (13).

$$x_{ij} = (l_{ij}, m_{ij}, n_{ij}, s_{ij})$$ (12)

$$(x_{ij})^{-1} = (s_{ij}^{-1}, n_{ij}^{-1}, m_{ij}^{-1}, l_{ij}^{-1})$$ (13)

The third step is to compute weight by performing summation and multiplication processes. The final answers will be used to compute weight, $w$. The computation of weight, $w$ for each variable is summarized in Table 1.
Fuzzy Analytic Hierarchy Process (FAHP) Integration for Decision Making Purposes: A Review

All the solutions will be grouped and represented by

$$w_j = (x_j \delta^{-1}, \beta_j \gamma^{-1}, \gamma_j \beta^{-1}, \delta_j \alpha^{-1})$$  \hspace{1cm} (14)

From the Equation (14), the result provides four answers but not in crisp value [18]. Therefore, the last step is to perform defuzzification. All four values are to be substituted into the following equation:

$$N = \frac{b+c + [(d-c)-(b-a)]}{2}$$

$$N = \frac{3b+3c+d-c-b+a}{6}$$

$$N = \frac{a+2b+2c+d}{6}$$  \hspace{1cm} (15)

After obtaining the crisp value for all criteria, the value is normalized to get the summation of one. This method is more recommended compared with both AHP and Triangular FAHP because of the use of 4 values representation for scaling purposes. The more values are used, the more accurate it will be while performing weight scaling.

### Table 1: Computation of weight, $w$[17]

<table>
<thead>
<tr>
<th>Variables</th>
<th>Product</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\alpha_j = \prod_{i=1}^{n} a_{ij}^{1/n}$</td>
<td>$\alpha = \sum_{j=1}^{n} \alpha_j$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\beta_j = \prod_{i=1}^{n} b_{ij}^{1/n}$</td>
<td>$\beta = \sum_{j=1}^{n} \beta_j$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\gamma_j = \prod_{i=1}^{n} c_{ij}^{1/n}$</td>
<td>$\gamma = \sum_{j=1}^{n} \gamma_j$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$\delta_j = \prod_{i=1}^{n} d_{ij}^{1/n}$</td>
<td>$\delta = \sum_{j=1}^{n} \delta_j$</td>
</tr>
</tbody>
</table>
2.2 Fusion of Fuzzy AHP to Obtain Rank or Score

Some fusions of Fuzzy and AHP are made to determine solution from a finite set of points. The chosen points are the “shortest” points based from previous FAHP to determine relative weight. “Closest” points are known as positive ideal and “further” points are considered as negative ideal solution [19]. Two methods typically used for ranking and scoring are Fuzzy TOPSIS and Fuzzy VIKOR.

2.2.1 Fuzzy Technique for Order Performance by Similarity to Ideal Solution (TOPSIS)

TOPSIS consists of eight steps. Most Fuzzy TOPSIS use triangular AHP with fuzzy preference weight [20]. It is expressed in the form of $\tilde{w}_i = (lw_i, mw_i, uw_i)$. The second step is to choose the appropriate linguistic judgment. The third step is to construct a matrix form. The matrix contains fuzzy numbers as presented in Equation (16).

$$D = \begin{bmatrix} C_1 & C_2 & \cdots & C_n \\ A_1 & x_{11} & x_{21} & \cdots & x_{1n} \\ A_2 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}, i = 1,2,\ldots,m; j = 1,2,\ldots,n$$

(16)

The fourth step is to perform normalization. The normalized values are calculated using

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1,2,\ldots,m; j = 1,2,\ldots,n$$

(17)

The fifth step is to identify the weighted normalized value ($v_{ij}$) which can be identified through $v_{ij} = \tilde{w}_i.r_{ij}$. Weight is symbolized as $(\tilde{w}_i)$ whereby the weight of j-th attribute. Next stage is to determine the ideal solution either to be positive or negative. This is basically the sixth step in Fuzzy TOPSIS. Fuzzy positive ideal solution (FPIS) is represented by Equation (18) and as for negative ideal solution (FNIS) could be represented using Equation (19).

$$A^+ = (v_1^+, v_2^+, \ldots, v_n^+)\text{ where, } v_j^+ = (1,1,1)$$

(18)

$$A^- = (v_1^-, v_2^-, \ldots, v_n^-)\text{ where, } v_j^- = (0,0,0)$$

(19)
\[
d(A_i, A^+) = d(A_i, A^-) = \sqrt{\frac{1}{3}[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]}
\]

(20)

\[
CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, i = 1,2,...,m
\]

(21)

Distant is between each alternative from \(A^+\) and \(A^-\). The seventh step of this method is to determine distance between alternative through Equation (20) which will be represented in the form of \((d_i^+, d_i^-)\). The last step is to compute the coefficient closeness for each alternative by applying Equation (21). The closest final answer to 1 will be ranked and scored as the best alternatives to carry out as a decision. Equation (21) shows how to calculate the closeness coefficient. Fuzzy TOPSIS is different from the triangular and trapezoidal due to its capability to provide the best ranking.

2.2.2 Fuzzy VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)

The main step of Fuzzy VIKOR consists of 4 steps. The first task is to determine the best and worst value in fuzzy number form. Previous author [21] termed it as FPIS for the best and FNIS is presenting the worst fuzzy value. The second step is to compute the best and worst fuzzy value using the Equation (22). By obtaining the fuzzy positive and negative value, the values are substituted in Equations (23) and (24) to obtain \(S_i\) and \(R_i\). Calculating these values are the second step in fuzzy VIKOR.

\[
f_j^+ = \max x_{ij} \text{ and, } f_j^- = \min x_{ij}
\]

(22)

\[
S_i = \sum_{i=1}^{n} w_j \left( \frac{f_j^+ - f_{ij}}{f_j^+ - f_j^-} \right)
\]

(23)

\[
R_i = \max_j \left( w_j \left( \frac{f_j^+ - f_{ij}}{f_j^+ - f_j^-} \right) \right)
\]

(24)

\(w_j\) symbolize criterion weightage express expert’s decision making preference. \(S_i\) is known as \(A_i\) respect to criterion computed by distant summation for desired values. \(R_i\) is equivalent to \(A_i\) respectively towards \(j/\text{th alternative}\). \(A_i\) is used to compute obtaining highest distant...
from most positive value [22]. Step 3 is to determine the index value, \( Q_i \) as shown in Equation (25):

\[
Q_i = \frac{v(S_i - S^+)}{S^- - S^+} + \frac{(1-v)(R_i - R^+)}{R^- - R^+}
\]  

(25)

Compared to Fuzzy TOPSIS, VIKOR have shorter steps and easy to compute. It takes shorter time for computation and has the same function as Fuzzy TOPSIS. The last step for Fuzzy VIKOR is to create a table consists of values of \( S_i \), \( R_i \) and \( Q_i \) in ascending order for scoring purposes.

3.0 DISCUSSION

In this section, discussion on two different perspectives of Fuzzy AHP applications are conducted. Table 2 summarizes the implementation of Fuzzy AHP for selected engineering–related research domains that typically involve Multi–Criteria Decision Making (MCDM) problems. The studies proposed various enhanced method based on Fuzzy AHP method through theory expansion.

This includes the implementation in wide range of research areas such as Information and Communication Technology (ICT)–related problems, supply chain, construction or anything related to production. However, decision making in selecting variables for economic indicator applied in design for remanufacturing is hardly obtained. This is due to the parameters obtained are always linguistic [49]. Table 3 summarizes the applications of Fuzzy AHP in manufacturing engineering domain. Several problems related to product, system and worker are mentioned pertaining to manufacturing industries. Table 4 shows two different classes of FAHP. To determine relative weight, only two types of FAHP namely triangular and trapezoidal are used. Triangular takes longer time to compute due to extra steps compared with trapezoidal.

The difficulty level of triangular is high due to number of steps and rules to be adhered. This type of FAHP will be used again for obtaining the ranking or scoring. The least number of steps is Fuzzy VIKOR and the most steps is Triangular FAHP. Triangular FAHP consists of many steps due to Hamming’s distance formula which is similar to computational time. These FAHP correspond towards the number of steps. In terms of difficulty level, VIKOR shows low level of difficulty because the steps are short.
Table 2: Fuzzy AHP Implementation for Various Research Domains

<table>
<thead>
<tr>
<th>No.</th>
<th>Researchers</th>
<th>Methods/Techniques</th>
<th>Research Domains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Triangular Fuzzy AHP</td>
<td>Trapezoidal Fuzzy AHP</td>
</tr>
<tr>
<td>1</td>
<td>Bulut et al. [23]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Saaty and Tran [10]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Wang et al. [24]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Chang [11]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Lee and Seo [25]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Song et al. [26]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Taylan et al. [27]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Rezaei et al. [28]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Abdullah and Najib [29]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Zheng et al. [17]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Abdullah and Zulkifi [31]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Xiaoqiong et al. [32]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Celik et al. [18]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Taylan et al. [33]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Avikal et al. [34]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Shaw et al. [35]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Dincer and Hacioglu [36]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Ashtiani and Azgomi [37]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Awasthi and Kannan [20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Liu et al. [38]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Maity [39]</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Musani and Jemain [40]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Selected Applications of MCDM in Manufacturing Engineering

<table>
<thead>
<tr>
<th>No.</th>
<th>Researchers</th>
<th>Manufacturing Fields</th>
<th>Experiment Problem</th>
<th>Base MCDM Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sari [41]</td>
<td>Author proposed a Fuzzy Multi-Criteria Decision Model with Monte Carlo simulation to solve problem related to determine the best RFID solution provider. The result shows best solution provider could be determined based on their manufacturing process.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Dong and Liang [42]</td>
<td>TOPSIS was proposed to solve problem in measuring manufacturing performances. Highest performance of manufacturing production is selected based on quality, cost and flexibility.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sevkli et al. [43]</td>
<td>Authors proposed using two methods which are Triangular FAHP and TOPSIS to solve problem of selecting best supplier. The result shows the best supplier selected based on criteria delivery, quality and cost.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Jung [44]</td>
<td>Author proposed an integration of Fuzzy Analytic Hierarchy Process (AHP) and Goal Programming (GP) to obtain the solution by selecting best partners for production planning. The result shows best manufacturing industry partner selected for production planning.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Taha and Rostam [45]</td>
<td>Authors proposed using a hybrid Fuzzy AHP and PROMETHEE decision support system to solve problem selecting best machine tool for a Flexible Manufacturing Cell (FMC).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Safari et al. [46]</td>
<td>Authors proposed combination of FMEA and Fuzzy VIKOR to solve problem regarding risk of architecture in manufacturing enterprise. The method ranks Enterprise architecture risk factors with respect to a set of criteria.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Rani et al. [47]</td>
<td>Authors used two methods TOPSIS and VIKOR to solve problem regarding product specification contribute by operator’s performance. The result shows defects of product are minimized after highest human errors are minimized.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Noor et al. [48]</td>
<td>Authors used two methods namely Triangular Fuzzy AHP and Trapezoidal Fuzzy AHP to solve problem selecting best material for drinking water bottle. The result shows one material selected following all criteria for fabrication process of drinking water bottle.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Additionally, Table 4 shows comparative attributes of several methods. As stated by [49], the variables and parameters involved are linguistic-based and are hard to differentiate because those are measured qualitatively rather than quantitatively. This is consistent with detailed articles published earlier [10-11]. As far as the review is concerned, even though numerous variations of Fuzzy AHP have been proposed to solve MCDM problems, the method can still be improved as depicted in some recent researches [12,15,20,22].

<table>
<thead>
<tr>
<th>Method</th>
<th>Fusion of FAHP to Determine Relative Weight</th>
<th>Fusion of FAHP to Obtain Ranking or Scoring</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative Computational Time</td>
<td>Relative Level of Complexity</td>
</tr>
<tr>
<td>Triangular FAHP</td>
<td>Longest</td>
<td>High</td>
</tr>
<tr>
<td>Trapezoidal FAHP</td>
<td>Shortest</td>
<td>Low</td>
</tr>
</tbody>
</table>

### 4.0 CONCLUSION

In a nutshell, in the context of MCDM methods, theory expansion, IT related problems, construction, education in school, supply chain and production are some of the areas that inherit MCDM problems. This article reviews some of the recent and significant research that highlights the use of Fuzzy AHP to solve the problems. Additionally, there are new set of industrial problem characteristics that inherit the characteristics of MCDM. One of them is Sustainable Manufacturing especially Design for Remanufacturing that consists of MCDM and dynamic factors. Fuzzy AHP could be utilized as part of the integrated method to solve problems.

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